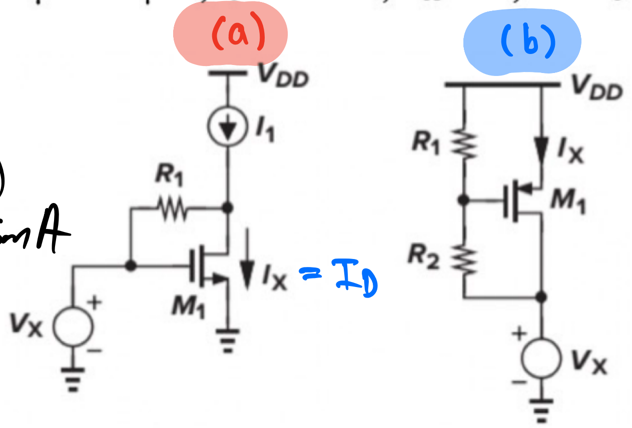
1. For each of the following circuits, determine the drain current and drain source voltage of the MOS transistor and its region of operation.
The transistors parameters are as follows:
NMOS: $V_{tn} = 0.5$ V, $\lambda_n = \gamma_n = 0$, $\mu_n C_{ox} = 200\ \mu A/V^2$.
PMOS: $|V_{tp}| = 0.5$ V, $\lambda_p = \gamma_p = 0$, $\mu_p C_{ox} = 100\ \mu A/V^2$.
$V_{DD} = 3$ V, $(W/L)_n = (W/L)_p = 5\ \mu m / 0.5\ \mu m$, $I_1 = 1.25$ mA, $V_X = 1$ V, and $R_1 = R_2 = 1\ k\Omega$

$I_B = I_1 - I_X$
$= 1.25mA - (I_D)$
$= 1.25mA - 0.25mA$
$= 1mA$



(a)

$$I_D = \frac{1}{2} \cdot 200 \times 10^{-6} \cdot \left(\frac{5}{0.5}\right)(1-0.5)^2$$

$= 250\mu A = 0.25mA$

$I_D = 0.25mA$

$$\frac{V_D - V_X}{R_1} = I_B$$

$$1mA = \frac{V_D - 1}{1k\Omega}$$

$\therefore V_D = 2V$

Check if in saturation:

$V_{DS} = 2V - 0V = 2V$

$2V > 1V - V_{th}$

$2V > 1V - 0.5V$ ✓

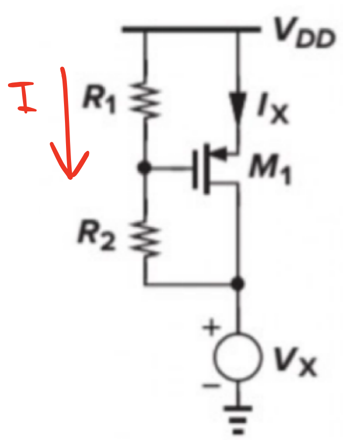
SATURATION

$I_D = 0.25mA$

$V_D = 2V$

SATURATION

(b)



$$I = \frac{V_{DD} - V_X}{R_1 + R_2} = \frac{3-1}{2k} = 1mA$$

$$V_G = V_{DD} - I_R R_1 = 3 - 1mA \cdot 1k\Omega = 2V$$

$$V_{SG} = V_S - V_G = 3V - 2V = 1V$$

$$I_D = \frac{1}{2} \cdot (100\mu) \cdot \left(\frac{5}{0.5}\right) \cdot (V_{SG} - |V_{thp}|)^2 = 0.125mA$$

$$V_D = V_X = 1V$$

$$V_{SD} = 3 - 1 = 2V$$

Check if in saturation :

$$V_{SD} > V_{SG} - |V_{thp}|$$

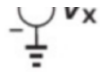
$$2V > 1V - 0.5V$$

SATURATION

$I_D = 0.125mA$

$V_D = 1V$

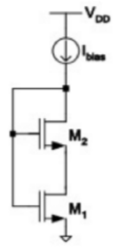
SATURATION



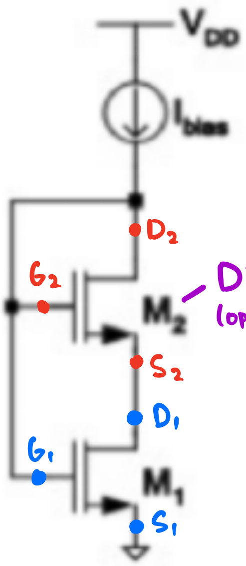
2. In the following circuit assume that the bulks of the two NMOS transistors are connected to ground, $\lambda=0$, and $\gamma\neq0$.

a) Assuming that $I_{bias} > 0$, what is the region of operation of transistor M_1?

b) Does the region of operation of M_1 depend on the relative sizing of the transistors. In other word, does it depend on the values W_1, W_2, L_1, and L_2.



(a)



it the bulks of the two NMOS transistors are connected to ground,

- $V_{S_1b_1} = V_{S_2b_2} = 0$

Diode connected (operating Saturation)

$V_{GS_2} > V_{th_2}$

$V_{GS_2} = V_{GS_1}$

$\therefore V_{GS_1} > V_{th_2}$

$V_{th_2} > V_{th_1}$

$V_{GD_1} > V_{th_1}$

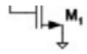
$V_{GD_1} - V_{th_1} > 0$

$V_{GS_1} - V_{th_1} > V_{DS_1}$

- So Triode as well

- The region of operation of transistor M_1 is **Triode Region**

(b) **NO**, it does not depend on the relative sizing of the transistor (W_1, L_1, W_2, L_2)



3. This question is based on Problem 2.13 of the textbook by Behzad Razavi: The transit frequency, f_T, of a MOS transistor is defined as the frequency at which the small-signal current gain of the transistor is equal to unity (while the source and drain terminals are held at ac ground).

(a) Show that:

$$f_T = \frac{g_m}{2\pi(C_{GS} + C_{GD})}$$

(b) Using square-law characteristics (long channel equations that we have seen in class), show that for an NMOS of size (W/L) that operates in saturation region we have

$$f_T \approx \frac{3\mu_n V_{eff}}{4\pi L^2}$$

This relation shows th dependence of speed of operation to the technology feature size and to the supply voltage.

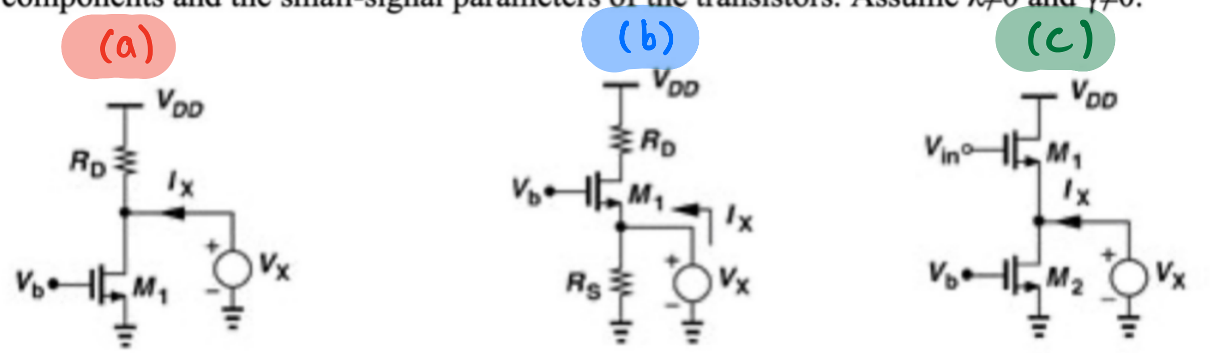
(c) Assuming that in the subthreshold region of operation the drain current of the device is given by:

$$I_D = I_0 e^{\frac{V_{GS}}{\eta V_T}}$$

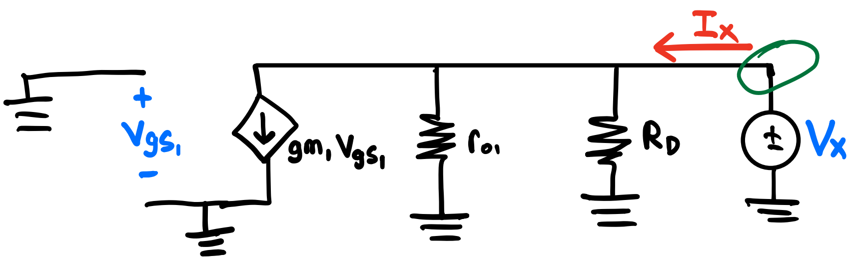
where $\eta \approx 1.5$ and V_T=kT/q (V_T=26mV at the room temperature), and using the equation in part (a) of this question, find an expression for the f_T of a MOS device when it is operating in the subthreshold region.

(d) Compare the result of part (c) with that of part (a) for the f_T of the same transistor and comment on the relative value of f_T of the transistor when it is operating in the active region compared to when it is in the subthreshold region.

4. Calculate the small-signal output resistance (Vx/Ix) of the following circuits based on the circuit components and the small-signal parameters of the transistors. Assume $\lambda \neq 0$ and $\gamma \neq 0$.



(a) Small Signal Model:



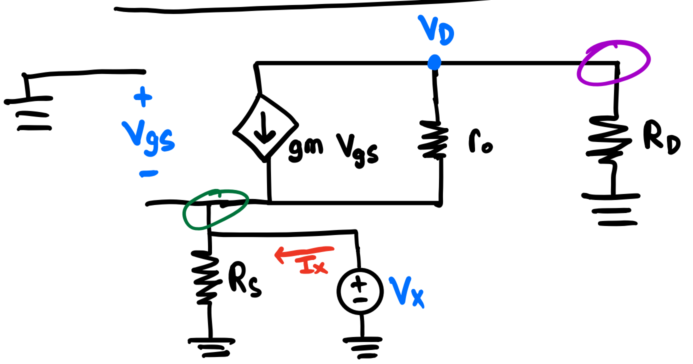
KCL: $I_x = \frac{V_x}{r_{o1}} + \frac{V_x}{R_D}$

$I_x = V_x\left(\frac{1}{r_{o1}} + \frac{1}{R_D}\right)$

$\frac{V_x}{I_x} = \left(\frac{1}{r_{o1}} + \frac{1}{R_D}\right)^{-1}$

$$\boxed{\frac{V_x}{I_x} = r_{o1} \parallel R_D}$$

(b) Small Signal Model:



KCL: $I_x + g_m V_{GS} + \frac{(V_D - V_x)}{r_o} = \frac{V_x}{R_S}$

KCL: $0 = \frac{V_D}{R_D} + g_m V_{GS} + \frac{(V_D - V_x)}{r_o}$

KVL: $V_{GS} = 0 - V_x$

$\frac{V_D}{R_D} - g_m V_x + \frac{V_D - V_x}{r_o} = 0$

$\frac{V_D}{R_D} = g_m V_x + \frac{V_x - V_D}{r_o}$

$\frac{V_D}{R_D} = g_m V_x + \frac{V_x}{r_o} - \frac{V_D}{r_o}$

$V_D\left(\frac{1}{R_D} + \frac{1}{r_o}\right) = V_x\left(g_m + \frac{1}{r_o}\right)$

$\therefore V_D = \frac{V_x\left(g_m + \frac{1}{r_o}\right)}{\frac{1}{R_D} + \frac{1}{r_o}}$

$$\therefore V_D = \frac{V_x\left(g_m + \frac{1}{r_o}\right)}{\frac{1}{R_D} + \frac{1}{r_o}}$$

Plug into KCl:

$$= \frac{V_x}{R_S} + g_m V_x + \frac{V_x - \frac{V_x\left(g_m + \frac{1}{r_o}\right)}{\frac{1}{R_D} + \frac{1}{r_o}}}{r_o}$$

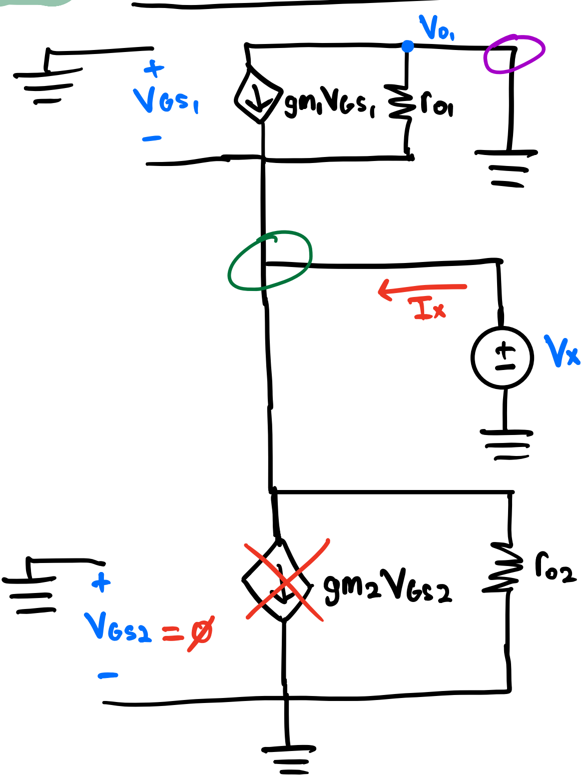
$$I_x - g_m V_x + \frac{V_D - V_x}{r_o} = \frac{V_x}{R_S}$$

$$I_x = \frac{V_x}{R_S} + g_m V_x + \frac{V_x - V_D}{r_o}$$

$$I_x = \frac{R_D V_x + R_S V_x g_m r_o + R_S V_x + V_x r_o}{R_D R_S + R_S r_o}$$

$$\boxed{\frac{V_x}{I_x} = \frac{R_D R_S + R_S r_o}{R_D + R_S + r_o + g_m R_S r_o}}$$

(c) Small Signal Model:



KCl₁: $g_{m1}V_{GS1} + I_x + \frac{V_{D1} - V_x}{r_{o1}} = \frac{V_x}{r_{o2}}$

KCl₂: $0 = g_{m1}V_{GS1} + \frac{V_{D1} - V_x}{r_{o1}}$

KVL: $V_{GS1} = -V_x$

$$\frac{V_{D1}}{r_{o1}} - \frac{V_x}{r_{o1}} = g_m V_x$$

$$\frac{V_{D1}}{r_{o1}} = g_m V_x + \frac{V_x}{r_{o1}}$$

$$V_{D1} = \left(g_m V_x + \frac{V_x}{r_{o1}}\right) r_{o1}$$

Plug into KCl₁:

$$I_x = \frac{V_x}{r_{o2}} + g_m V_x + \frac{V_x - (V_x + g_m V_x)}{r_{o1}}$$

$$= \frac{V_x}{r_{o2}} + g_m V_x + \frac{V_x}{r_{o1}}$$

$$I_x = \frac{V_x}{r_{o2}} + g_m V_x + \frac{V_x}{r_{o1}}$$

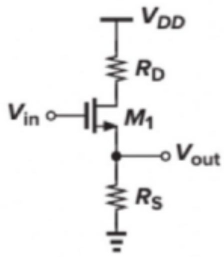
$$\frac{V_x}{I_x} = \left(\frac{1}{r_{o2}} + g_m + \frac{1}{r_{o1}}\right)^{-1}$$

$$\boxed{\frac{V_x}{I_x} = \frac{r_{o1} r_{o2}}{g_m r_{o1} r_{o2} + r_{o1} + r_{o2}}}$$

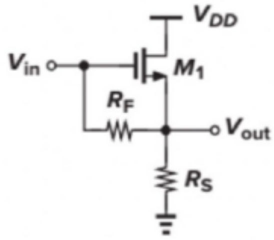
No Body Effect

5. In each of the following circuits, assuming that the transistor is operating in the saturation region and $\lambda_n = \gamma_n = 0$, find the small-signal gain (V_{out}/V_{in}) as a function of circuit components and the transistor's small-signal parameters.

(a)

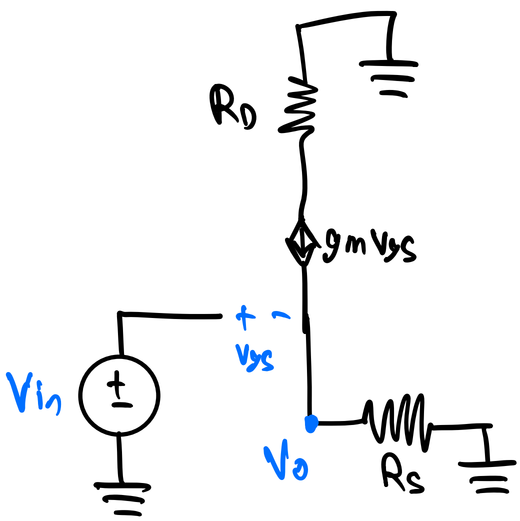


(b)



(a)

Small Signal Model:



$$V_{gs} = V_{in} - V_o$$

$$V_o = g_m V_{gs} R_S$$

$$\therefore V_o = g_m [V_{in} - V_o] R_S$$

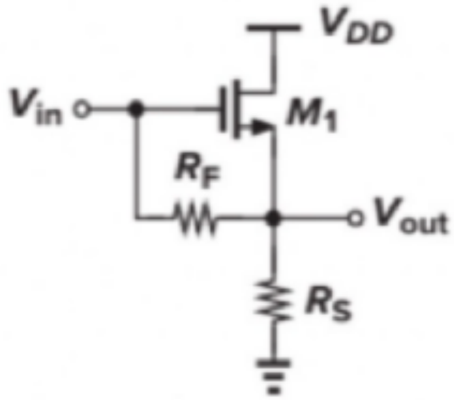
$$V_o = g_m V_{in} R_S - g_m V_o R_S$$

$$V_o + g_m V_o R_S = g_m V_{in} R_S$$

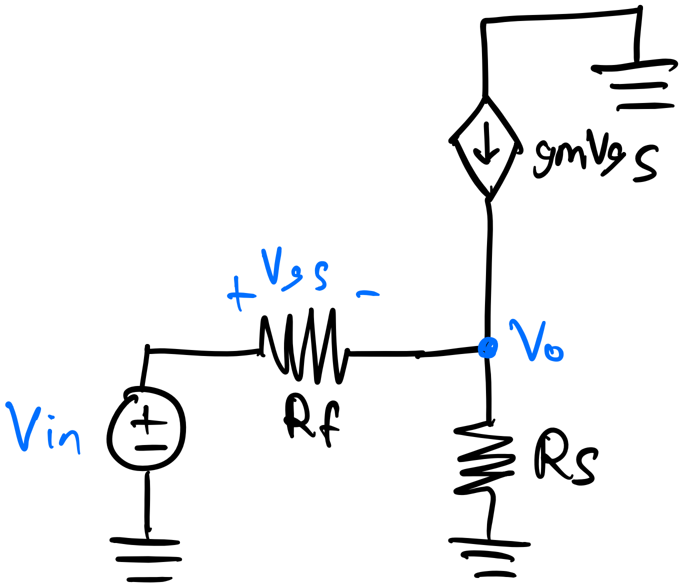
$$V_o (1 + g_m R_S) = g_m V_{in} R_S$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{g_m R_S}{1 + g_m R_S}}$$

(b)



Small Signal Model:



$$V_{in} - V_o = V_{GS}$$

$$\frac{V_{in}-V_o}{R_f} + g_m[V_{in}-V_o] = \frac{V_o}{R_s}$$

$$\frac{V_{in}}{R_f} + \frac{V_o}{R_f} + g_m V_{in} - g_m V_o = \frac{V_o}{R_s}$$

$$\frac{V_o}{R_s} - \frac{V_o}{R_f} + g_m V_o = \frac{V_{in}}{R_f} + g_m V_{in}$$

$$\frac{V_o}{V_{in}} = \frac{\frac{1}{R_f} + g_m}{\frac{1}{R_s} + \frac{1}{R_f} + g_m} = \frac{R_s + R_s R_f g_m}{R_f + R_s + R_f R_s g_m}$$